Institut für Signalverarbeitung und Spachkommunikation, Technische Universität Graz

Exam for Fundamentals of Digital Communications (2VO) on 20-5-2016

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Duration: 3 Stunden
Permitted material: Table of Fourier transform properties, tabulated Q-function (both are provided), pocket calulator, and equation handbook. **The assignment sheets and provided tables must be returned in the end of the exam!**

Theory 1 (30 Points)

(a) Discuss the differences between a binary antipodal and a binary orthogonal modulation scheme. Which one performs better (over an addtive white Gaussian noise (AWGN) channel) and how can this be explained?
(b) The maximum a-posteriori probability (MAP) criterion is the most generic decision rule for minimizing the error probability in a digital communication system. Which conditions must be given that (i) the minimum distance metric: $\arg\min_m ||\mathbf{r} - \mathbf{s}_m||^2$; and (ii) the *following*, specialized correlation metric: $\arg\max_m(\mathbf{r}^T\mathbf{s}_m)$ can be used? The vector $\mathbf{r}$ denotes the received signal vector; $\{\mathbf{s}_m\}$ are the transmitted signal vectors.
(c) Explain the projection of a vector in $\mathbb{R}^3$ onto a subspace in $\mathbb{R}^2$. What does the projection theorem state?

Problem 1 (30 Points)

In a binary digital communication system, two signal waveforms are transmitted, which are defined by the signal vectors

$$\mathbf{s}_1 = \begin{bmatrix} \sqrt{\mathcal{E}_b} \\ 0 \end{bmatrix} \quad \text{and} \quad \mathbf{s}_2 = \begin{bmatrix} 0 \\ \sqrt{\mathcal{E}_b} \end{bmatrix},$$

in a signal space given by the basis functions

$$\psi_1(t) = \begin{cases} 1 & \text{for } 0 \le t \le 1 \\ 0 & \text{elsewhere} \end{cases} \quad \text{and} \quad \psi_2(t) = \begin{cases} 1 & \text{for } 0 \le t \le 1/2 \\ -1 & \text{for } 1/2 < t \le 1 \\ 0 & \text{elsewhere} \end{cases}.$$

(a) Sketch the two signal waveforms.
(b) Compute the energies of these signals, the angle between the signal vectors, and determine the distance between the signal vectors.
(c) Sketch the correlation type demodulator for these signals, based on the given signal space representation. Explain, why this is an optimal receiver structure for an AWGN channel.

(d) The signals are transmitted at equal a-priori probabilities over an AWGN channel with noise power spectral density $S_n(f) = N_0/2$. Determine the decision rule for optimal detection (obtaining minimum error probability).
(e) Compute the error probability for a signal-to-noise ratio per bit of $\mathcal{E}_b/N_0 = 8 = 9$ dB.
(f) Show an alternative correlation type demodulator that is based on an alternative signal space, such that only one correlator is needed for the decision. Determine and sketch the impulse response of this matched filter.

Problem 2 (25 Points)

A random variable X has a Gaussian PDF with mean $m_X = 0$ and variance $\sigma_X^2 = 1$ and random variable Y has the PDF

$$f_Y(y) = 0.3\delta(y - s_1) + 0.7\delta(y - s_2),$$

which can be interpreted as the probability density function of a binary transmission system with symbols $s_1 = -1$ and $s_2 = 1$ (with prior symbol probabilities $P(s_1) = 0.3$ and $P(s_2) = 0.7$.
(a) Illustrate the PDFs of both random variables X and Y.
(b) Compute mean, variance, and second moment for the random variable Y.
(c) Assume a third random variable is defined as $Z = X + Y$. Illustrate the PDF of Z.
(d) Compute mean, variance, and second moment for the random variable Z. Use the moments of the random variables X and Y to get these results.
(e) Compute the conditional PDF $f_{Z|Y}(z|y = s_1)$ for the random variable Z, given that random variable $Y = s_1$.
(f) Given the realization $Z = z$, find a threshold γ for an optimum decision whether $Y = s_1$ or $Y = s_2$, according to the maximum a-posteriori (MAP) criterion.
(g) Determine the error probability (=probability of wrong detection) for the MAP decision.

Problem 3 (15 Points)

The signal $x(t)$ is applied to a linear, time-invariant system with impulse response $h(t)$, where

$$x(t) = \delta(t) + \sigma(t) - \sigma(t-2), \qquad \sigma(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1 & \text{for } t \geq 0 \end{cases},$$

$$h(t) = \begin{cases} e^{-t/2} & \text{for } t \geq 0 \\ 0 & \text{elsewhere} \end{cases},$$

and $\delta(t)$ is the Dirac pulse.
(a) Sketch the signals $x(t)$ and $h(t)$. The convolution integral can be used to determine the output signal $y(t)$ of the LTI system. Define this convolution integral in its generic form.
(b) Plugging the functions $x(t)$ and $h(t)$ into the convolution integral, define and sketch the convolution integrals for the following time intervals:

$$\begin{aligned} 1: &\quad t < 0 \\ 2: &\quad 0 \leq t \leq 2 \\ 3: &\quad t > 2 \end{aligned}$$

(c) Determine and sketch the output signal $y(t)$.