Tracking Algorithms for Multipath-Aided Indoor Localization

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Presentation overview

- Virtual-anchor based localization
- Statistical modeling of influences on multipath-aided localization
- Tracking algorithms to enhance robustness
- Performance results and discussion
Virtual Anchor based Localization – Scenario

Scenario

- Room, floor plan given
- One single anchor node (known location)
- Range-based localization
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- Signal reaches agent also via reflections
- Reflections mapped to anchors outside room
- Location computable → virtual anchors (VAs)
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How to localize?

- Usage of UWB signals
- Multipath components resolvable
- Perform ranging to each VA

Multipath extraction

- Extract $N$ pseudoranges $z_1, \ldots, z_N$, possible errors:
  1. Mapping $z_i$ to VAs
  2. False positive detections
  3. Obstructed, ”invisible” VAs

Figure: UWB-CIR, BW 7.5 GHz
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Statistical model for ranges and ML estimation (1)

- A model accounting for uncertainties in $z_i$ is

$$p_{z_i|p}(z_i|p) = P_{VA} \sum_k v_k \mathcal{N}(z_i | ||p-p_k||, \sigma_k^2) + (1-P_{VA})p_{z_i,\overline{VA}}(z_i)$$

- $p$ ... Position variable
- $P_{VA}$ ... Prob. that $z_i$ corresponds to VA
- $v_k$ ... Prob. that k-th VA is visible
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Statistical model for ranges and ML estimation (2)

→ ... multimodality, straightforward ML leads to large outliers

- ML estimator maximizes the objective

\[
\hat{p}_{ML} = \arg \max_{\mathcal{p}} p_{z|p}(z|p) = \arg \max_{\mathcal{p}} \prod_{i=1}^{N} p_{z_i|\mathcal{p}}(z_i|\mathcal{p})
\]
How to obtain an accurate and robust estimator?

- Introduce position prior PDF and perform MAP estimation

\[ \hat{p}_{\text{MAP}} = \arg \max_p p(p)p_z|_p(z|p) \]

- Mitigation of the multimodality

- Prior information by propagation of position estimate from one time step to the next for a moving agent

- Use a standard state-space model

\[ x_{k+1} = f(x_k, w_k) \]
\[ y_k = h(x_k, v_k) \]

- Include statistical models in these equations
Moving agent

- Agent moves along trajectory
- Characterized by motion model

\[
x_{k+1} = \begin{bmatrix} 1 & 0 & \Delta T & 0 \\ 0 & 1 & 0 & \Delta T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x_k + w_k
\]

- Models e.g. pedestrian motion
- Prior knowledge by correlation of successive positions and finite agent velocity

![Figure: Example trajectory and velocities](image-url)
State-space model and estimation – Noise models

- Probabilistic description of observation equation
  \[ y_k = h(x_k, v_k) \] via PDFs of noise \( v_k \):

- Two possibilities for measurements \( y_k \)
  - ML-estimates: Heavy-tail distribution \( \rightarrow \) Model: Bivariate symmetric Cauchy distribution
  - Pseudoranges: No deterministic linear \( h(x_k, v_k) \) \( \rightarrow \) Model: Likelihood function \( p_{z|\mu}(z|\mu) \)

\( \rightarrow \) Both issues render a Kalman filter useless
Modifying the Kalman filter to alleviate outliers

- ML-estimates as $y_k \rightarrow$ KF is heavily influenced
- Assume $\hat{x}_{k-1}^+$ is "good"
- Next position probably in vicinity $\rightarrow$ prior knowledge

**Figure:** Trajectory and ML estimates
Modifying the Kalman filter to alleviate outliers

- ML-estimates as $y_k \rightarrow$ KF is heavily influenced
- Assume $\hat{x}^+_{k-1}$ is "good"
- Next position probably in vicinity $\rightarrow$ prior knowledge
- Place Gaussian position prior $p_k(p)$ over predicted position

Figure: $k \rightarrow k + 1$ (Prediction)
Modifying the Kalman filter to alleviate outliers

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- Refine the ML-measurement:

$$y_k = \arg \max_p p_{z|p}(z|p)$$

Figure: log-likelihood of $z_k$
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$$y_k = \arg \max_p p_k(p)p_z|p(z|p)$$

- Finally, Kalman update
KF with measurement refinement – Performance (example)

Figure: Performance of KF, perfect initialization, refinement makes KF a robust estimator, parameter tuning effort reduced
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Initialization – Gaussian sum filter (GSF)

- Previous KF variant assumes good estimate at $k - 1$
- Initialization? E.g. at doors
- GSF: $M$ parallel (refined) KFs
- Posterior PDF of $x_k$ mixture of $M$ Gaussians

Figure: Initialization of KF
Initialization – Gaussian sum filter (GSF)

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- KFs can compute "responsibility" for current measurement
- Weights of all but one KF decay quickly

Figure: GSF initialization
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Figure: First 10 ML-estimates
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Figure: Weights of KFs
Gaussian sum filter – Performance (example)

Figure: Performance of GSF, random init with $M = 10$
Gaussian sum filter – Performance (example)

Figure: Performance of GSF, random init with $M = 10$, zoom
Bayesian State estimation – Particle filters

- KF-based schemes can not fully account for statistical model
  - $y_k$ ... ML-estimate: Cauchy distribution
  - $y_k$ ... Pseudorange-vector: Likelihood function $p_{z|p}(z|p)$

→ Particle filter for Bayesian state estimation

- Initialization: Select set of initial particles, equal weight
- Prediction: Propagate each particle via state equation
- Update: Likelihood $p(y_k|\text{particle})$ for measurement
  1. ML-estimates: Bivariate Cauchy distr. centered at ML-est.
  2. Pseudoranges: Value of $p_{z|p}(z|p)$ at particle
- Resampling: Posterior particles drawn from predicted ones according to likelihoods
- Compute estimate from particles (e.g. $x$ and $y$ median)
Example – Particle filter with pseudoranges
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[Graphs showing particle movements and distribution]
Example – Particle filter with pseudoranges
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![Diagram showing particle filter results with pseudoranges.]

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Example – Particle filter with pseudoranges

- Diagram 1: Path of the particle filter over time in a 2D space with x and y coordinates.
- Diagram 2: Probability density map showing the distribution of the particles.
- Diagram 3: Relative number of particles distribution along px.
- Diagram 4: Relative number of particles distribution along py.

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- Particle filter with pseudoranges.
- Trajectory and probability distributions for different positions.
- Rel. # particles in different positions.

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Example – Particle filter with pseudoranges
Example – Particle filter with pseudoranges

![Diagram showing particle filter results with pseudoranges in a 2D space and corresponding histograms for relative number of particles in x and y dimensions.](image)
Particle filters – Performance (example)

Figure: Performance of PF with pseudorange measurements
Particle filters – Performance (example)

Figure: Performance of PF with ML measurements, Cauchy model
All estimators – Performance comparison

Figure: Average performance for 50 runs over trajectory
All estimators – Performance – Correlated Visibilities

- We expect VA-visibilities to have correlated behavior
  - KFs drawn away from true trajectory (prior is wrong)
  - PFs show less sensitivity
Conclusions and ongoing work

- Concept for multipath-aided localization and tracking
- Dealing with multimodal/heavy-tail models in our concept
- Variants of state-space estimators to gain robustness
- Effective use of floor plans and signal reflections

Ongoing work

- Multipath extraction from UWB impulse response
- Validation/extension of models with measurements
- Tracking of VAs

Thanks for your attention!