Narrowband Systems – Principle of Diversity and MIMO Systems

Klaus Witrisal

Signal Processing and Speech Communication Lab
Technical University Graz, Austria

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Outline

- **What is MIMO?**
  - **Error Rate in Fading Channels**
  - **Multiple Antennas in Wireless**
- Channel and Signal Models
- Spatial Diversity
- Space-Time (ST) Coding
- Summary

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References

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  - Figures copied from this reference

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Motivation: Error Rate in Fading Channels

- AWGN channel (BPSK, QPSK)

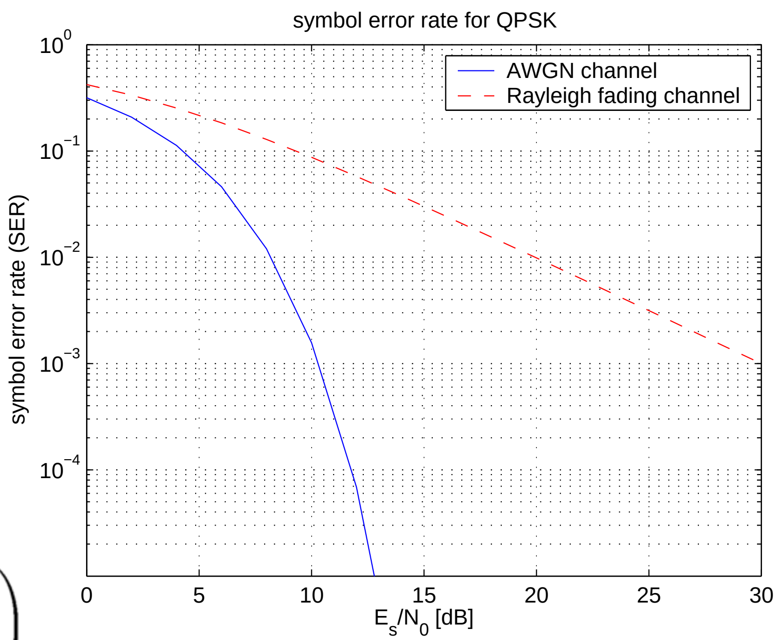
$$P_e(\gamma_b) = Q\left(\sqrt{2\gamma_b}\right)$$

- Fading channel
  - Received signal

$$r[k] = hs[k] + n[k]$$

  - $|h|$ is Rayleigh
  - Average error rate

$$\overline{P_e} = \frac{1}{2}\left(1 - \sqrt{\frac{\overline{\gamma_b}}{1+\overline{\gamma_b}}}\right)$$



What is MIMO?

- Application of Multiple Antennas
  (at the Transmitter **and**/or Receiver)
  to improve the link performance:

- Coverage (range)
- Quality
- Interference Reduction
- Spectral Efficiency

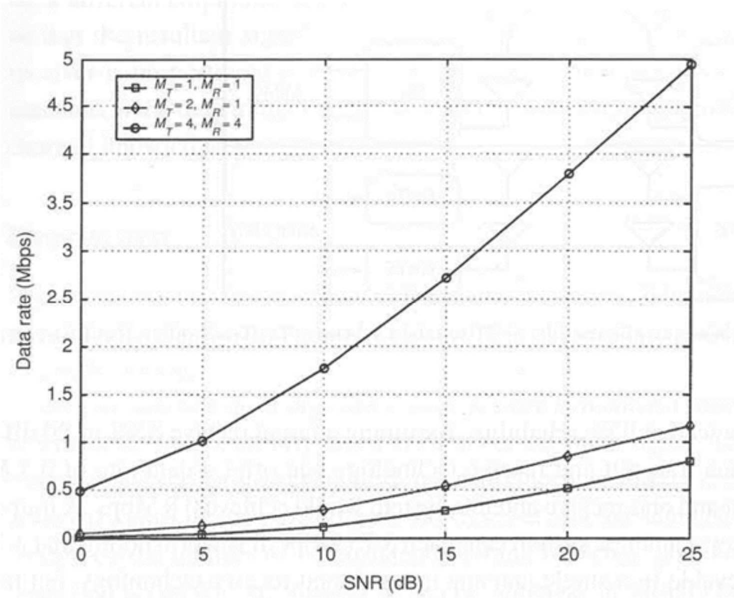
Multiple Antennas in Wireless - History

- Non-adaptive:
  - Directive antenna arrays (Marconi 1900)

- Adaptive:
  - Interference suppression by beam steering (military: 70's, 80's)
  - Receiver ST-Techniques to support co-channel signals: 90's
  - TX-RX ST-Techniques: 2000

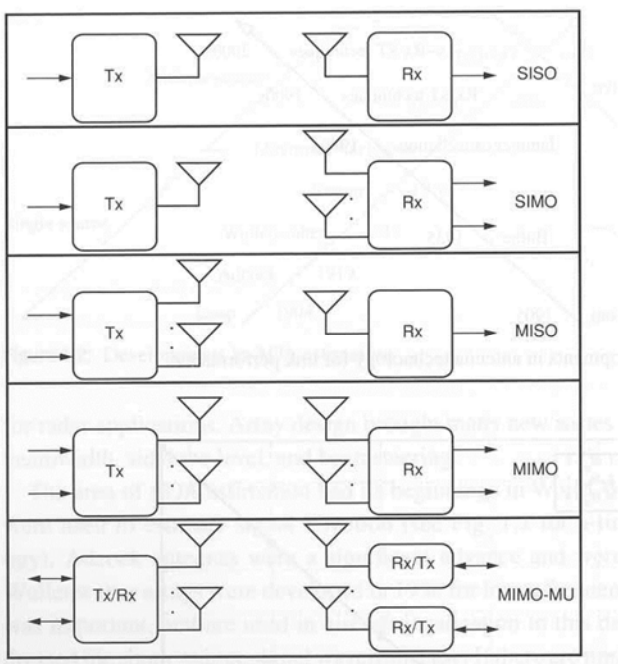
Multiple Antennas in Wireless – Potential

- Data rate at 95% reliability in a 200 kHz **fading channel**
- At SNR = 20 dB:
  - SISO: 0.5 MBit/s
  - 2 TX, 1 RX: 0.8 MBit/s
  - 4 TX, 4 RX: 3.75 MBit/s



Antenna Configurations

- Number of TX-antennas: M_T
- Number of RX-antennas: M_R
- MIMO channels can be exploited in several ways



Exploiting Multiple Antennas – Array Gain

- Array Gain:
  - **Average increase in SNR** due to coherent combining (at TX / RX or both) → beamforming
  - Average increase in SNR at RX is prop. M_R
  - MISO/MIMO (if $M_T > 1$): Channel knowledge required at TX to obtain array gain

Exploiting Multiple Antennas – Diversity Gain (1)

- Fading channel: variations of signal power
  - **Diversity** is used to combat fading (PDF of fading amplitude is changed)
- Receive antenna diversity (SIMO)
- **Diversity order**:
  - number of **independently** fading branches
  - In SIMO: number of RX antennas (if independent)

Exploiting Multiple Antennas – Diversity Gain (2)

- Transmit Diversity (MISO)
  - Possible with and without channel knowledge

- Space-time (ST) diversity coding:
  - applies coding across space to extract diversity **without** channel knowledge
  - Diversity order M_T (if channels are independent)

- MIMO: combination of Tx- and Rx-Diversity
  - Diversity order: $M_T M_R$

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Exploiting Multiple Antennas – Spatial Multiplexing (SM)

- Linear (in min(M_T,M_R)) increase in rate or capacity
  - no additional bandwidth
  - no additional power

- Requires MIMO-channels
- Multiplexing
  - Divide bit stream in several sub-streams
  - Transmit those from each antenna
  - Receiver can extract both streams **knowing channels**

  → increase of rate prop. number of antenna pairs (example on blackboard)
- For multi-users: MIMO-MU, SDMA

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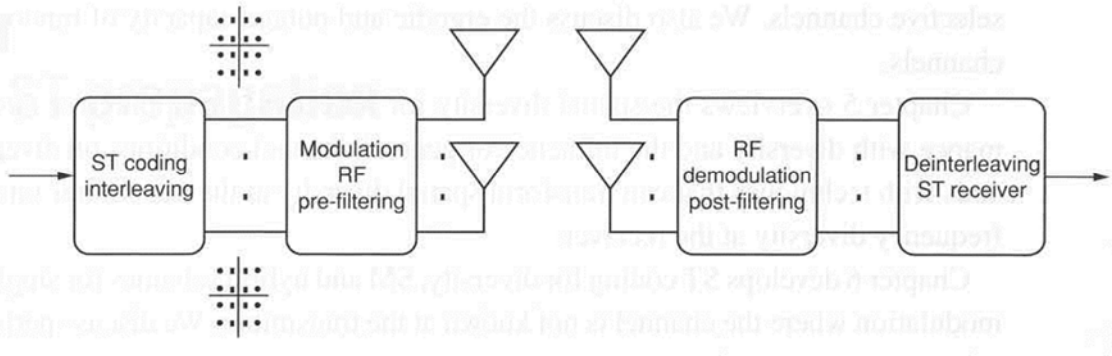
Exploiting Multiple Antennas – Interference Reduction

- Co-channel interference due to frequency-reuse
  - With multiple antennas, spatial signature of desired signal can be used to reduce interference
  - Requires channel knowledge
  - Can also be applied at TX (don't send to co-channel users)
- Exploiting multiple antennas:
  - It is generally **not possible** to achieve all goals simultaneously

ST Wireless Communications System



- Multiple antennas
- ST encoding and interleaving
- ST pre- and post-filtering
- ST decoding and de-interleaving

Outline

- What is MIMO?
- **Channel and Signal Models**
  - **Narrowband**
  - (Wideband)
- Spatial Diversity
- Space-Time (ST) Coding
- Summary

Fading (Small-Scale, Microscopic)

- **Multipath**:
  - Superposition of large number of scattered waves
    - Various magnitudes and phases
  - Re- and Im-components (of complex phasors) add up to **complex Gaussian** (by CLT)
- Amplitude distribution:
  - **Rayleigh** fading:
    - **Mean values** of Re/Im-components are **zero**
  - Ricean fading:
    - Dominant component
    - K-factor: power in dominant /power in scattered rays

Channel Variability

- Time Variability – Doppler Spread
  - Coherence time and Doppler spread: $T_C = 1/\nu_{rms}$
- Frequency Selectivity – Delay Spread
  - Coherence BW and RMS Delay spread: $B_C = 1/\tau_{rms}$
- Space Selective Fading – Angle Spread
  - Coherence distance and Angle spread: $D_C = 1/\theta_{rms}$
  - Doppler/Delay/Angle power spectra: average power as a function of …

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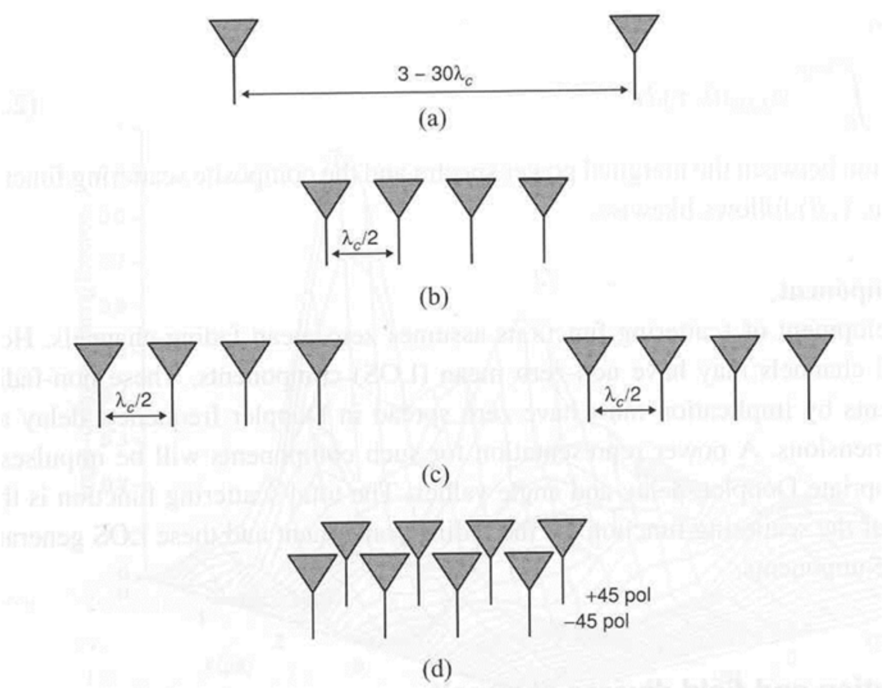
Array Topologies



Signal Models

- Input output relation
- Classifications:
  - SISO, SIMO, MISO, MIMO
  - Continuous time – discrete time (sampled)
  - **Frequency flat channel** ($T_s >> \tau_{rms}$ or $B << B_C$) – frequency selective channel ($B\tau_{rms} > 0.1$)

(we focus on **narrowband** systems → **flat fading**)

- For sampled signal model (single carrier), normalizations are introduced:
  - Bandwidth = 1 Hz, symbol period = 1 s

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Sampled Signal Models (1) – Math

- Frequency selective case (SISO)

$$y[k] = \sum_l \sqrt{E_s} s[l] h[k-l] + n[k]$$

- Frequency flat case (SISO) – channel is complex gain

$$y[k] = \sqrt{E_s} h s[k] + n[k]$$

- Frequency flat case (MIMO) – vector notation

$$\mathbf{y}[k] = \sqrt{\frac{E_s}{M_T}} \mathbf{Hs}[k] + \mathbf{n}[k]$$

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Sampled Signal Model (2) – SISO

- $h[k]$ … T_s-spaced sampled channel
  - $l = 0, 1, \ldots, L-1$; L … channel length in samples
  - complex equivalent baseband channel; incorporates:
    - physical channel, pulse-shaping at TX, matched filter on RX, sampling delay
- $s[k]$ … symbols to be transmitted
  - scalar linear modulation: PAM, QAM
- $n[k]$ … noise samples
  - assumed white ZMCSCG (zero-mean circular symmetric complex Gaussian) noise; $\mathrm{var}\{n[k]\} = \sigma_n^2 = N_0$
- $y[k]$ … received signal

Sampled Signal Model (3) - Normalizations

- Channel
  - Channel in **frequency flat channels**: $\mathrm{E}\{|h|^2\} = 1$
  - Rayleigh case: h is ZMCSCG (zero-mean circular symmetric complex Gaussian)
  - Multipath channels: total average energy of all taps = 1
- Signal
  - Signal energy: average transmit symbol energy (= power, since T_s = 1 s) E_s
  - MIMO, MISO: energy per symbol per antenna E_s/M_T
  - data are IID with zero mean, unit average energy symbol constellations
- Noise
  - noise power σ_n^2 = noise PSD N_0 due to B = 1 Hz

Sampled Signal Model (4) – MIMO (1)

Drop time-index *k*

$$\mathbf{y} = \sqrt{\frac{E_s}{M_T}}\mathbf{H}\mathbf{s} + \mathbf{n}$$

with

$\mathbf{y} = [y_1, y_2, ..., y_{M_R}]^T$ … $M_R \times 1$ received signal vector

$\mathbf{s} = [s_1, s_2, ..., s_{M_T}]^T$ … $M_T \times 1$ transmitted signal vector

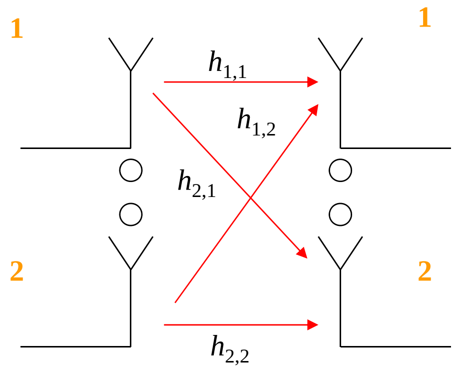
$\mathbf{n} = [n_1, n_2, ..., n_{M_R}]^T$ … $M_R \times 1$ noise vector

$\mathbf{H}$ … $M_R \times M_T$ channel matrix (complex channel gains)

noise: $n_i \sim (0, \sigma_n^2 = N_0)$, i.e. $E\{\mathbf{n}\mathbf{n}^H\} = N_0 \mathbf{I}_{M_R}$

Sampled Signal Model (5) – MIMO (2)

$$\mathbf{H} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix}$$



- Frequency-flat channel
  - Channel impact expressed by complex factors: channel transfer matrix
- **H** = **H**$_w$ is often assumed IID (spatially white channel)
  - in rich scattering

Statistical Properties of **H** - background

- Singular values of **H**
  - **H** has rank r
  - SVD: $\mathbf{H} = \mathbf{U\Sigma V}^H$: M_R x M_T
  - **U**: M_R x r
  - **V**: M_T x r
  - $\mathbf{\Sigma} = \text{diag}\{\sigma_1\ \sigma_2 \ldots\ \sigma_r\}$ (singular values)

- Eigen-decomposition of $\mathbf{HH}^H = \mathbf{Q\Lambda Q}^H$
  - $\mathbf{\Lambda} = \text{diag}\{\lambda_1\ \lambda_2 \ldots\ \lambda_r\}$

$$\lambda_i = \begin{cases} \sigma_i^2 & i = 1, 2, \ldots, r \\ 0 & i > r \end{cases}$$

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Squared Frobenius Norm of **H**

- Definition

$$\|\mathbf{H}\|_F^2 = \text{Tr}(\mathbf{HH}^H) = \sum_{i=1}^{M_R} \sum_{j=1}^{M_T} |h_{i,j}|^2$$

  - Interpretation: **total power gain of channel**
  - Using EV decomposition:

$$\|\mathbf{H}\|_F^2 = \sum_{i=1}^{M_R} \lambda_i$$

- PDF of power gain, when $\mathbf{H} = \mathbf{H}_w$ (IID channel)
  - chi-square distribution with $2M_T M_R$ degrees of freedom

$$f(x) = \frac{x^{M_T M_R - 1}}{(M_R M_T - 1)!} e^{-x} \sigma(x)$$

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(Wideband channels)

- Single carrier systems:
  - MIMO channel consists of channel impulse responses $h_{i,j}(\tau)$
  - Received signal is convolution with channels
  - MIMO system requires equalization → different fading at different delay taps can be exploited (RAKE receiver)

Outline

- What is MIMO?
- Channel and Signal Models
- **Spatial Diversity**
  - **Diversity gain**
- Space-Time (ST) Coding
- Summary

Diversity Gain (1)

- Wireless links are impaired by fading
- Diversity:
  - combine multiple **branches**; ideally uncorrelated
  - → reduce probability for deep fades
  - Condition for independence: separation > B_C, T_C, D_C
- Signal/symbol s sent over M branches:

$$y_i = \sqrt{\frac{E_s}{M}} h_i s + n_i, \qquad i = 1, \ldots M$$

E_s/M … symbol energy/branch
h_i … channel gain factor
n_i … ZMCSCG noise

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Diversity Gain (2)

- Maximum ratio combining

$$z = \sum_{i=1}^{M} h_i^* y_i$$

… derivation on blackboard …

- Upper bound on average symbol error rate for large SNR

$$\overline{P_e} \le \overline{N_e} \left(\frac{\rho d_{\min}^2}{4M} \right)^{-M}$$

N_e … number of nearest neighbors
d_{min} … their separation distance
$\rho = E_s/N_0$ … SISO average SNR

- → Diversity affects slope of SER curve

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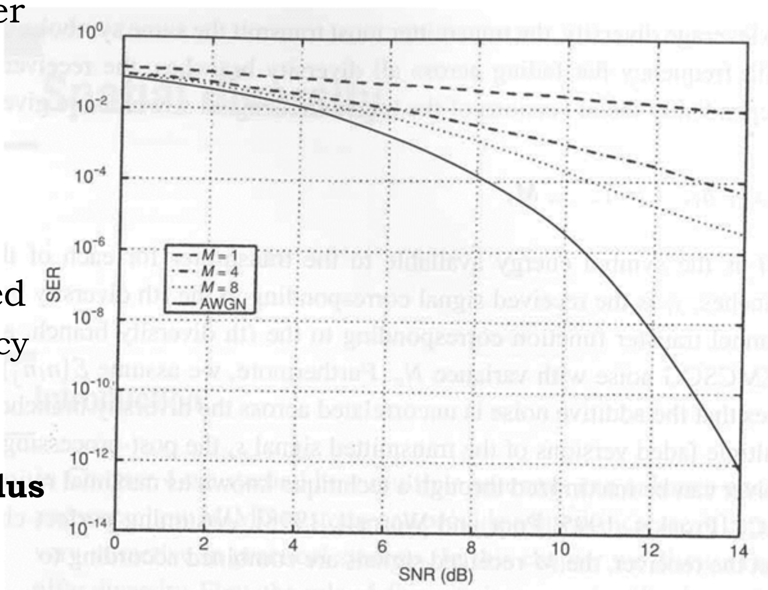
Diversity Gain (3)

- For infinite diversity order
  - AWGN performance is approached
  - → blackboard
- Here: repetition code used
  - Loss in spectral efficiency
  - AWGN: coding gain
  - Fading: diversity gain **plus** coding gain



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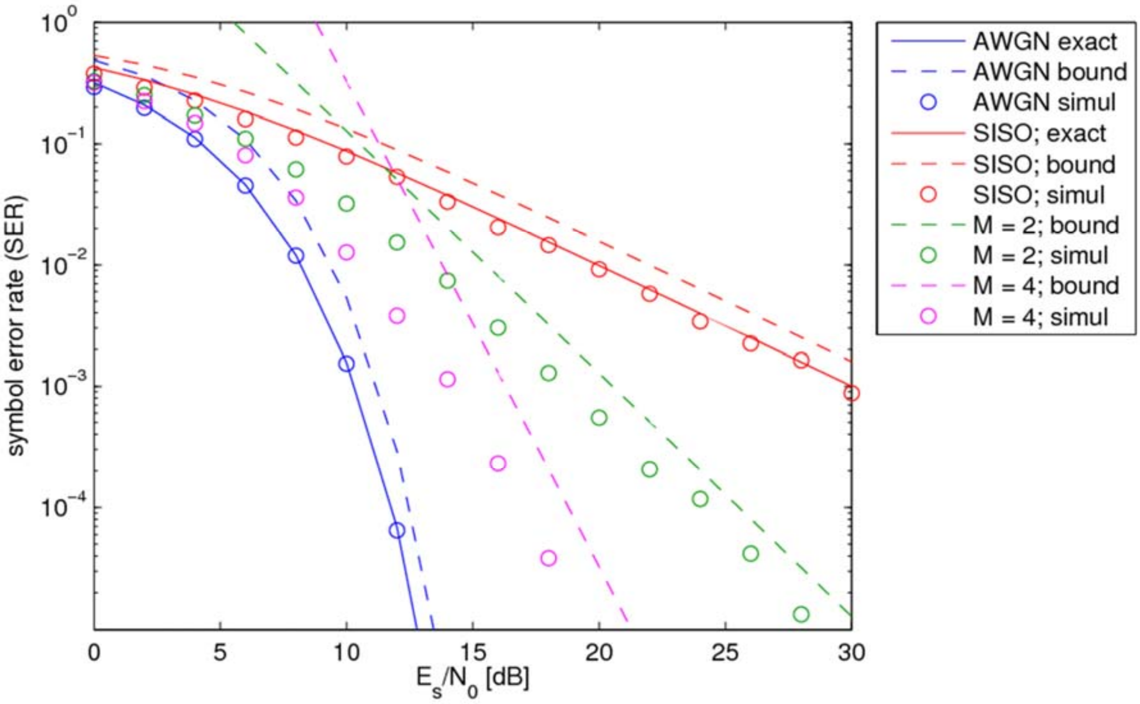
Diversity Gain (4)



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Coding Gain vs. Diversity Gain

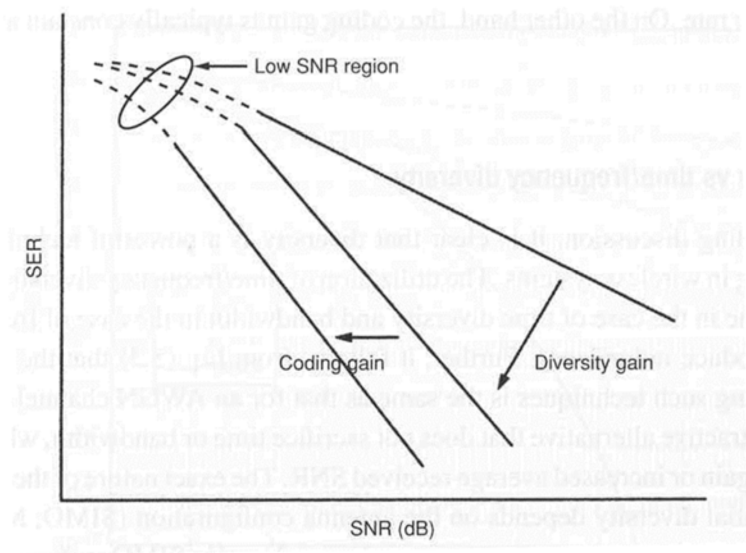
- Approx. equation

$$\overline{P}_e \approx \frac{c}{(\gamma_c \rho)^M}$$

c … constant; modulation and channel

$\gamma_c \geq 1$ … coding gain, array gain

M … diversity order



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Spatial Diversity vs. Time or Frequency Diversity

- Spatial diversity
  - No additional bandwidth required
  - Increase of average SNR is possible
  - Additional array gain is possible

  - These benefits are NOT possible with time or frequency diversity

- Diversity techniques
  - Depend on antenna configuration (SIMO, MISO, MIMO)

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Receive Antenna Diversity

- Assume flat fading

  $\mathbf{y} = \sqrt{E_s}\mathbf{h}s + \mathbf{n}, \ \mathbf{h} = [h_1 \ h_2 \ \dots \ h_{M_R}]^T$ channel vec.
- Maximum ratio combining
  - Assume perfect channel knowledge at receiver
  - Assume independent fading
  - SER at high SNR: $\overline{P_e} \leq \overline{N_e} \left(\frac{\rho d_{min}^2}{4} \right)^{-M_R}$
  - Diversity gain M_R
- Average SNR at RX: $\overline{\eta} = M_R \rho$
  - array gain M_R, 10 log M_R [dB]

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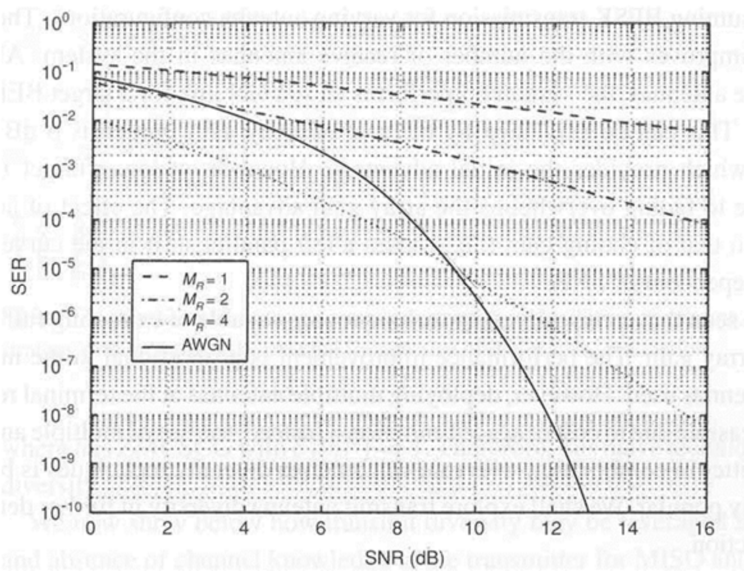
Receive Antenna Diversity – Performance

- Can be better than AWGN due to array gain
- At low BER fading disadvantage dominates

→ **full diversity and array gain** (prop. M_R) is achieved with receive diversity!



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Transmit antenna diversity

- Why is pre-processing needed?
  - Signal s is transmitted at ½ power from two antennas

$$y = \sqrt{\frac{E_s}{2}}(h_1 + h_2)s + n$$

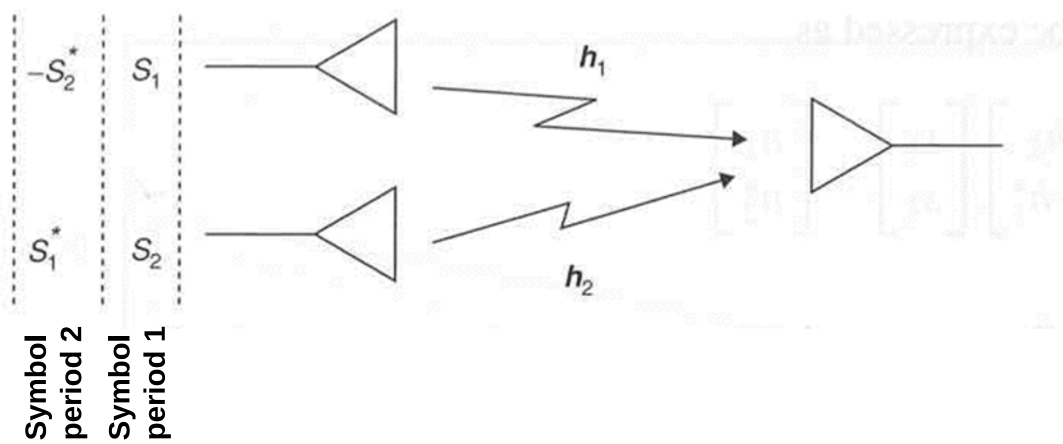
  - h_1 and h_2 are unit variance ZM complex Gaussian
- Equivalent signal model

$$y = \sqrt{E_s}\,hs + n$$

  - h is also unit variance ZM complex Gaussian!
  - → **NO diversity**

Alamouti Scheme, MISO

- Simple but ingenious method of pre-processing
- Channel is **unknown** to the transmitter



Alamouti Scheme – Derivation of Performance

- Channel
  - Frequency-flat
  - Constant over two symbol periods

sym. per. 1: $y_1 = \sqrt{\frac{E_s}{2}} h_1 s_1 + \sqrt{\frac{E_s}{2}} h_2 s_2 + n_1$

sym. per. 2: $y_2 = -\sqrt{\frac{E_s}{2}} h_1 s_2^* + \sqrt{\frac{E_s}{2}} h_2 s_1^* + n_2$

receiver:

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \sqrt{\frac{E_s}{2}} \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix} = \sqrt{\frac{E_s}{2}} \mathbf{H}_{\text{eff}} \mathbf{s} + \mathbf{n}$$

$$\mathbf{z} = \mathbf{H}_{\text{eff}}^H \mathbf{y} = \sqrt{\frac{E_s}{2}} ||\mathbf{h}||^2 \mathbf{I} \mathbf{s} + \tilde{\mathbf{n}}$$

$$z_i = \sqrt{\frac{E_s}{2}} ||\mathbf{h}||^2 s_i + \tilde{n}_i, \text{ for } i = 1, 2$$

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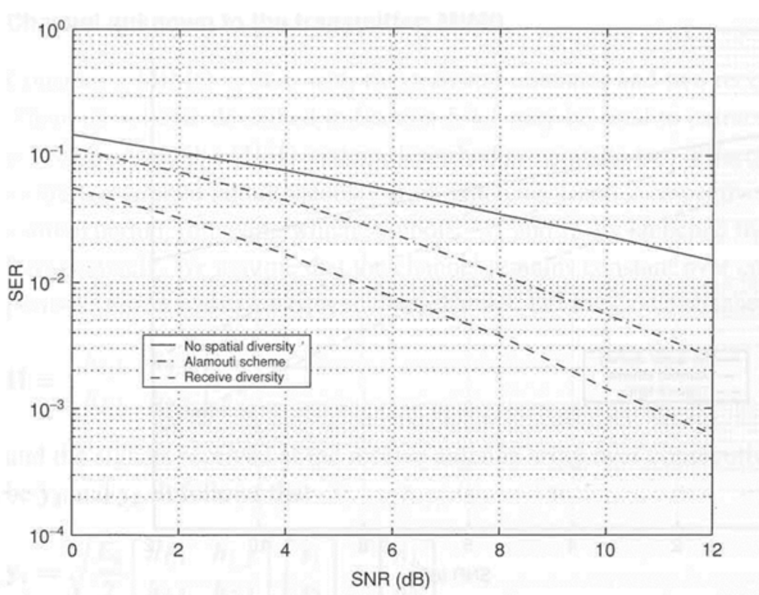
Alamouti Scheme - Performance

- Full $M_T = 2$ diversity
- Average SNR at receiver not increased

$$\bar{\eta} = \rho$$

→ no array gain!



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TX-Diversity – Channel Known

- Transmit weighted signals: $s_i = w_i s$
- **Goal:** symbols should arrive in phase
  - Vector channel: $\mathbf{h}^T = [h_1 \ h_2 \ \ldots \ h_{M_T}]$
  - Signal at receiver: $y = \sqrt{E_s}\mathbf{h}^T\mathbf{w}s + n$
  - Optimum weight vector: $\mathbf{w} = \frac{\mathbf{h}^*}{||\mathbf{h}||}$
- **Transmit MRC combining**
  - Derivation of array and diversity gain

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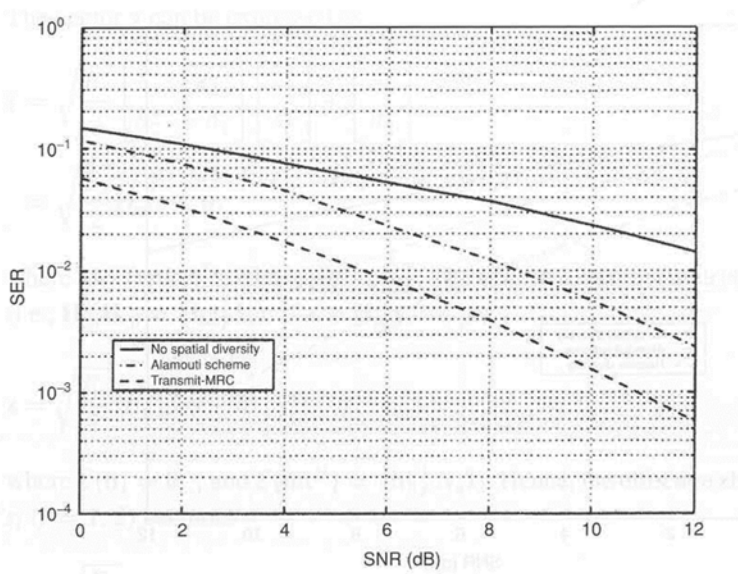
Transmit MRC Combining - Performance

- Diversity order M_T
- Array gain: M_T

→ equivalent to receive MRC

- Problem: Channel must be known at TX

- Figure:
  - Alamouti vs. TX-MRC



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Alamouti – Extension to MIMO

- MIMO scheme for $M_T = 2$; channel unknown
- Transmitted symbols: Like MISO Alamouti
  - Channel Matrix: $\mathbf{H} = \begin{pmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{pmatrix}$

  Receiver stacks two consecutive received symbols

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} = \sqrt{\frac{E_s}{2}} \mathbf{H}_{eff} \mathbf{s} + \mathbf{n}$$

  - $\mathbf{H}_{\text{eff}}$ is orthogonal!

$$\mathbf{H}_{\text{eff}}^H \mathbf{H}_{\text{eff}} = ||\mathbf{H}_{\text{eff}}||_F^2 \mathbf{I}_2 = \sum_{k=1}^{2} \sum_{l=1}^{2} |h_{k,l}|^2 \mathbf{I}_2$$

MIMO with Unknown Channel – Performance Limits

- Assume $\mathbf{H} = \mathbf{H}_w$; high SNR range
  - average SER: $\overline{P}_e \leq \overline{N}_e \left(\frac{\rho d_{\min}^2}{4M_R} \right)^{-2M_R}$

  → Diversity order: $M_T M_R = 2M_R$

  - average SNR: $\overline{\eta} = M_R \rho$

  → Only receive array gain!

MIMO: Channel Known to Transmitter

- "Dominant Eigenmode Transmission"
- transmitted signal: one s weighted by $\mathbf{w}$
  - like MISO
- received signal vector

$$\mathbf{y} = \sqrt{\frac{E_s}{M_T}}\mathbf{H}\mathbf{w}s + \mathbf{n}, \quad \|\mathbf{w}\|_F^2 = M_T$$

  - form a weighed sum: $z = \mathbf{g}^H\mathbf{y}$
  - to maximize SNR at the receiver → blackboard

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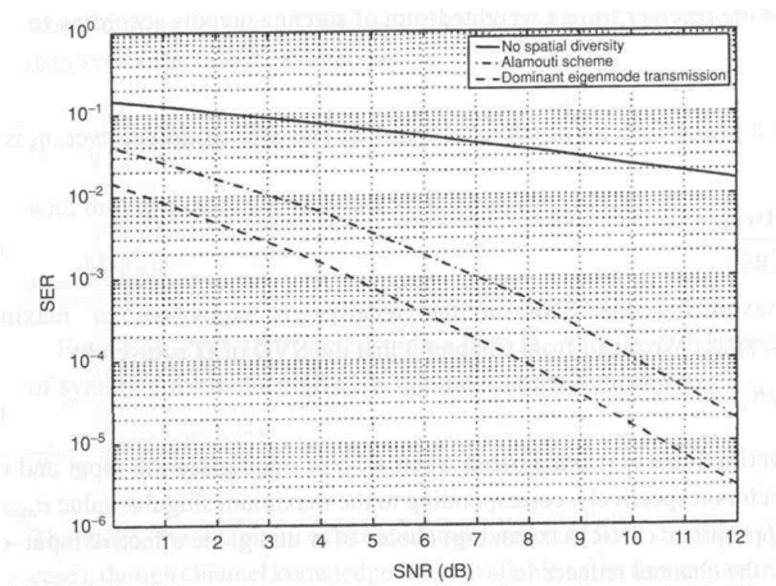
MIMO, Channel Know - Performance

- Diversity order: $M_T M_R$
- array gain: $E\{\lambda_{max}\}$, bounded by $\max(M_T, M_R)$ and $M_T M_R$
- Figure:
  - Dominant EM vs. Alamouti; 2x2
  - Same slope
  - different array gain



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Summary – Diversity Order

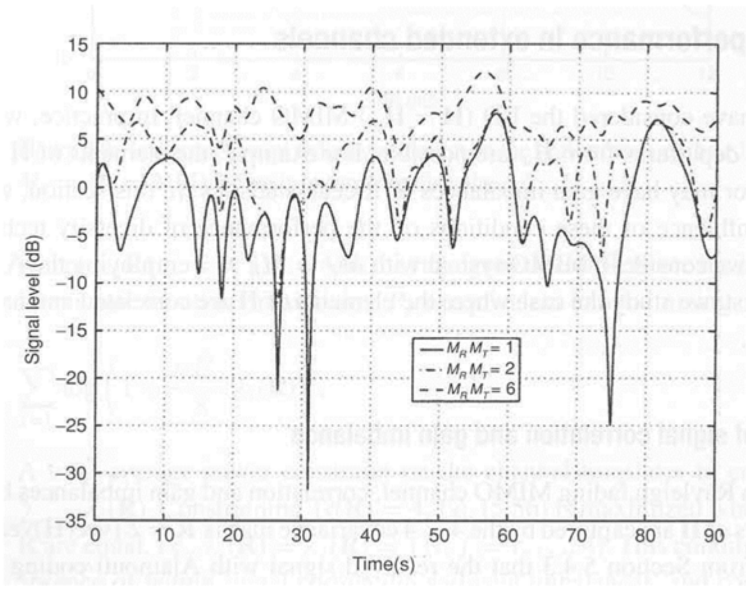
Configuration	Exp. array gain	Diversity order
SIMO (CU)	M_R	M_R
SIMO (CK)	M_R	M_R
MISO (CU)	1	M_T
MISO (CK)	M_T	M_T
MIMO (CU)	M_R	$M_R M_T$
MIMO (CK)	$\max(M_T, M_R) \leq E\{\lambda_{max}\} \leq M_T M_R$	$M_R M_T$

Channel Variability

- May be quantified by coefficient of variability

$$\mu_{\text{var}} = \frac{1}{\sqrt{M_T M_R}}$$

- AWGN case is approached if $M_R M_T \rightarrow \infty$, i.e., $\mu_{var} \rightarrow 0$



Diversity Order in Extended Channels

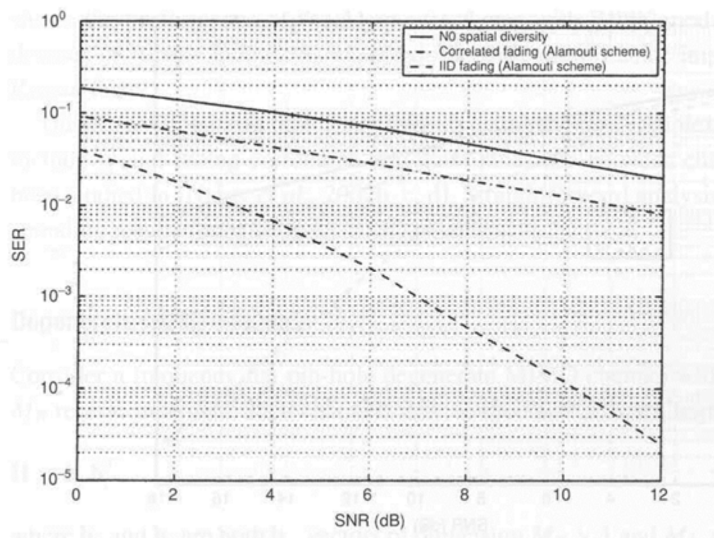
- The channel matrix **H** is not $\mathbf{H}_w$:
  - Elements of **H** are correlated
  - Elements of **H** have gain imbalances
  - Elements of **H** have Ricean amplitude characteristics
- Here: consider impact on Alamouti 2 x 2

Influence of Signal Correlation

- Diversity order decreases to $r(\mathbf{R})$, where **R** is the (4 x 4) covariance Matrix:

  $\mathbf{R} = E\{vec(\mathbf{H})vec(\mathbf{H})^H\}$

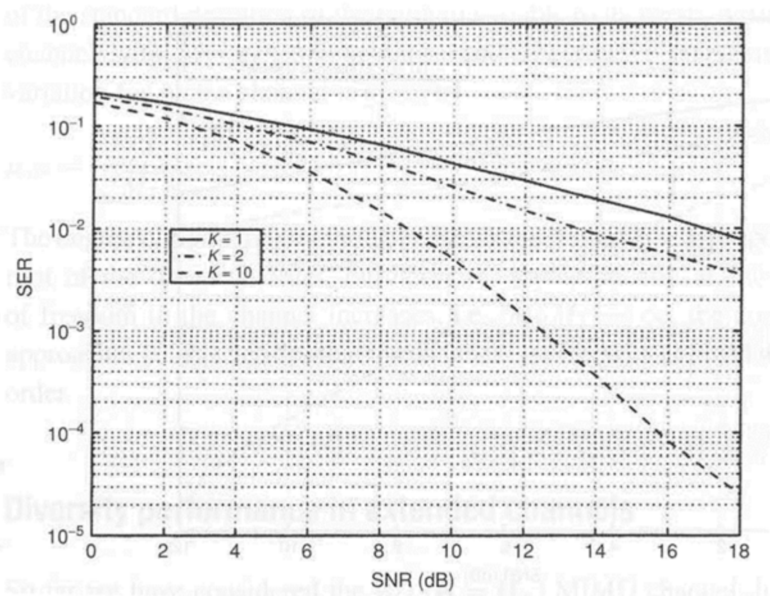
- Figure:
  - elements of **H** are fully correlated → $r(\mathbf{R}) = 1$
  - only array gain is present
  - no diversity gain



Influence of Ricean Fading

- The LOS component stabilizes the link

→ performance improvement with increasing K



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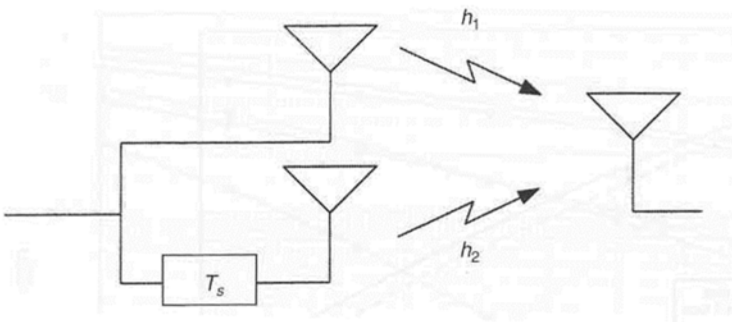
Indirect Transmit Diversity (1)

- **Delay diversity**
  - delay is one symbol interval
- Flat MISO channel is translated into two-path SISO channel (symbol spaced)

→ ML-detector can capture second-order diversity



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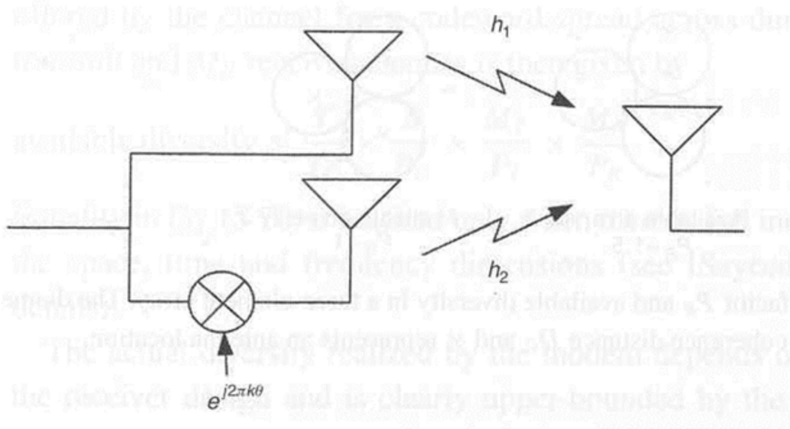
Indirect Transmit Diversity (2)

- **Phase-roll diversity**
- Effective channel at a certain time-separation is uncorrelated
- FEC and time-interleaving has to be used to exploit this



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Diversity of a Space-Time-Frequency Selective Channel

- four “dimensions” are available to exploit diversity:
  - nb. transmit antennas (M_T) (space 1)
  - nb. receive antennas (M_R) (space 2)
  - duration of the codeword (time)
  - signal bandwidth (frequency)
- available diversity gain depends on ratios of these parameters to the coherence-bandwidth, -time, -distance (packing factor)

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Space-Time coding

- Use coding across space and time to optimize the link performance
  - diversity gain (upper bounded by $M_T M_R$ if $\mathbf{H}_w$)
  - array gain (upper bounded by M_R if CU or $M_R M_T$ if CK)
  - coding gain (depends on min. distance of the code)

- Also: how to realize $M_T > 2$

- In frequency selective channels:
  - frequency diversity can be exploited

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Summary

- Multiple Antennas to improve link performance:
  - Coverage (range)
  - Quality
  - Interference Reduction
  - Spectral Efficiency

- Exploiting Multiple Antennas
  - Array Gain
  - Diversity Gain
  - Spatial Multiplexing
  - Interference Reduction

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