Mobile Radio Systems – OPAM: Understanding OFDM and Spread Spectrum

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Outline

- Introduction – Signal space representation of communications signals
- Generalization (of OFDM) to orthogonal pulse amplitude modulation (OPAM)
- Spread Spectrum
- Code-Division Multiplexing

References

- J. R. Barry, E. A. Lee, D. G. Messerschmitt: *Digital Communication*, 3rd ed., 2004, Kluwer
- J. G. Proakis and M. Salehi: *Communication Systems Engineering*, 2nd ed., 2002, Prentice Hall
- J. G. Proakis: *Digital Communications*, 4th ed., 2000, McGraw Hill

Figures extracted from these references

Signal Spaces (1)

- Representation of signals in a **linear** vector space
  - Allows geometric interpretations (distance, angle, etc.)
  - **Linear vector algebra** can be used for analysis and signal processing
  - Applies to **continuous** signals and **discrete** signals "seamlessly"!

Signal Spaces (2)

What is a signal space?

- In digital communications:
  - Information is transmitted by choosing an element from a set of M waveforms $\{s_m(t)\}$, $m = 1, 2, ...M$
  - Signal space of these M waveforms

$$\mathcal{S} = \text{span}\{s_1(t), s_2(t), ..., s_M(t)\}$$

set of **all signals** that can be represented by **linear combiniations** of these M waveforms

Signal Spaces (3)

Orthonormal basis of a signal space

- **minimum** set of N **orthonormal functions** that can express the elements $s(t) \in \mathcal{S}$

$$s(t) = \sum_{i=1}^{N} s_i \psi_i(t)$$

- orthonormal (basis) functions

$$\int_{-\infty}^{\infty} \psi_i(t)\psi_k^*(t)dt = \delta[i-k] = \begin{cases} 1 & i = k \\ 0 & i \neq k \end{cases} \rightarrow ||\psi_i(t)|| = 1$$

Signal Spaces (4)

- Set of M communication waveforms $\{s_m(t)\}$, $m = 1, 2, ...M$ can be equivalently **expressed as vectors** in this signal space

$$\mathbf{s}_m = [s_{m,1}, s_{m,2}, ..., s_{m,N}]^T$$

$$s_{m,i} = \int_{-\infty}^{\infty} s_m(t)\psi_i^*(t)dt = \langle s_m(t), \psi_i(t)\rangle$$

- Linear operations on continuous (or discrete) signals can be expressed as **linear vector operations**
- AWGN channel: received signal projected onto signal space **"sufficient statistic"**

$$r(t) = s_m(t) + n(t)$$

$$\mathbf{r} = \mathbf{s}_m + \mathbf{n}$$

Signal Spaces (5)

Correlation demodulator (filters matched to basis functions); output is received signal vector

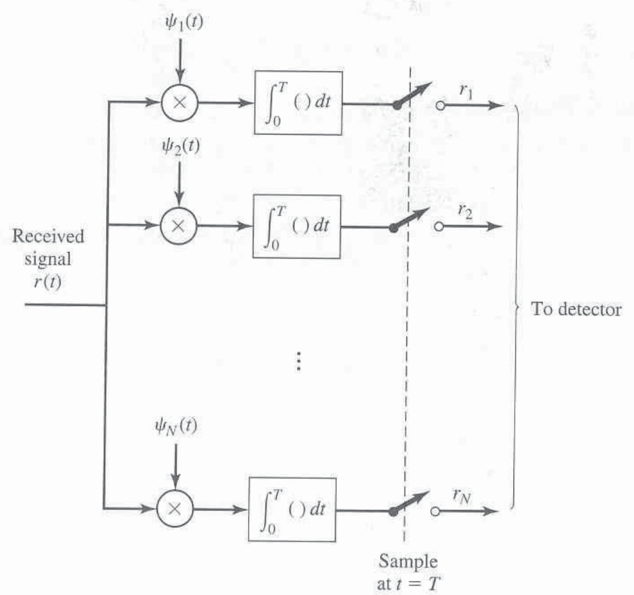


Figure 7.31 Correlation-type demodulator.

Signal Spaces (6)

Characterization of noise vector

- mean

$$\mathrm{E}\{n_i\} = ... = \int_0^T \mathrm{E}\{n(t)\}\psi_i(t)\,dt = 0$$

- (co)-variance

$$\mathrm{E}\{n_i n_j\} = ... = \frac{N_0}{2}\delta[i-j]$$

- i.e.: N noise components are **zero-mean, uncorrelated, Gaussian (i.i.d.) random variables** with variance $\sigma_n^2 = N_0/2$
- Error prob. for binary antipodal PAM $P_b = Q(\sqrt{2E_b/N_0})$

OPAM (1)

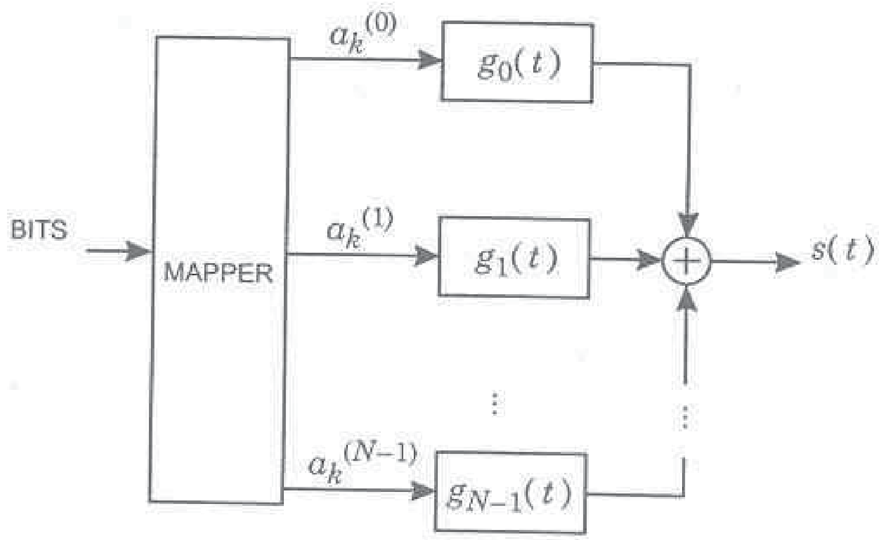
- **Orthogonal modulation**: (e.g. FSK)
  - M waveforms $\{s_m(t)\}$, $m = 1, 2, ...M$ are orthogonal
  - spectral efficiency: $\propto \log_2(M)/M$ bit/Hz/s
- **Pulse amplitude modulation** (PAM)
  - one basis function ($N = 1$) expresses M waveforms
  - spectral efficiency: $\propto \log_2(M)$ bit/Hz/s
- **OPAM combines both**

$$s(t) = \sum_{k=-\infty}^{\infty} \sum_{n=0}^{N-1} a_k^{(n)} g_n(t-kT)$$

N orthogonal pulse shapes $\{g_n(t) : n = 0, ..., N-1\}$ are amplitude modulated by different symbols $a_k^{(n)}$

OPAM (2)

- OPAM transmitter



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OPAM (3)

Examples for OPAM (and specializations)

- PAM: $N = 1$
- QAM: $N = 2$ (complex, 2-dimensional symbols)

$$g_0(t) = \sqrt{2}\cos(2\pi f_c t)g(t)$$

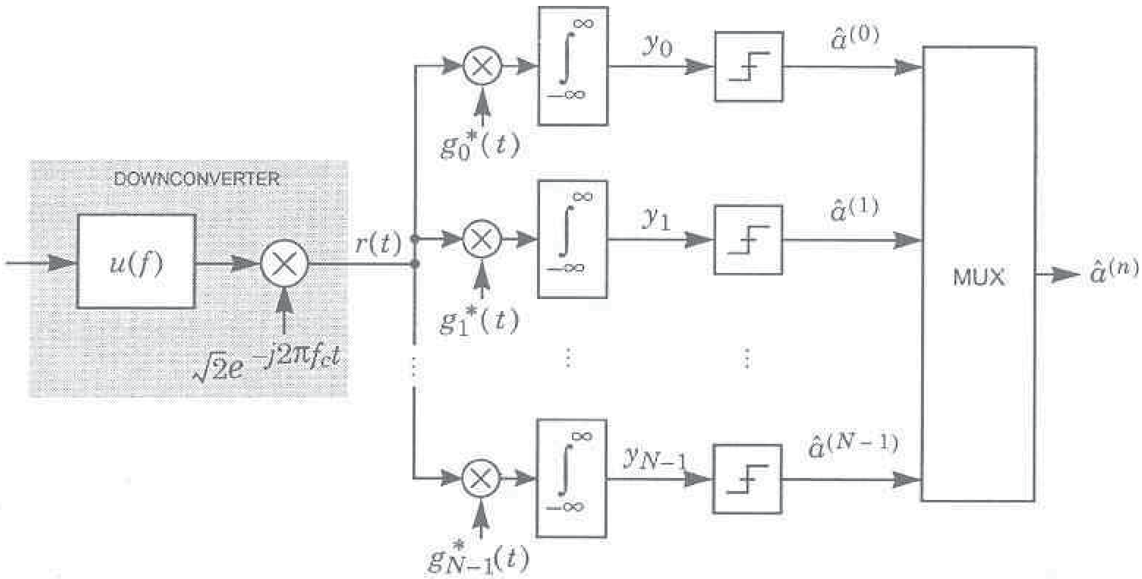
$$g_1(t) = -\sqrt{2}\sin(2\pi f_c t)g(t)$$

- orthogonal signaling: $a_k^{(n)} \in \{0, 1\}$; only **one** of $\{a_k^{(0)}, a_k^{(1)}, ..., a_k^{(N-1)}\}$ has value 1 at a time
- OFDM: $a_k^{(n)}$ are (2-dimensional) QAM symbols

$$g_n(t) = \frac{1}{\sqrt{T}} e^{j2\pi nt/T} w(t) \qquad \text{for } n = 0, 1, ..., N/2 - 1$$

OPAM (4)

- OPAM receiver: correlation demodulation / bank of matched filters



OPAM (5) – Spread Spectrum (1)

- In OFDM, basis functions are well localized in frequency
- Dimensionality of signal space:
  - PAM: $N = 1$; time-bandwidth product $WT \geq 1/2$
  - QAM: $N = 2$; $WT \geq 1$
  - OFDM: dimensionality $N = 2N_{SC} \approx 2WT$

  minimum required bandwidth: $W \geq N/(2T)$
- **Spread spectrum** (PAM) signals *deliberately* **use more than minimum (Nyquist) bandwidth**: $W \gg 1/(2T)$
  - SNR with matched filter (after **de-spreading**) is independent of pulse-shape
  - thus **error probability** (vs. E_b/N_0) **is not affected!**
  - but input SNR is reduced: $P_b = Q(\sqrt{2WT \cdot \mathrm{SNR}})$

Spread Spectrum (2)

Reasons for using a large bandwidth

- less sensitive to channel impairments (**frequency-selective multipath fading**)
  - **RAKE receiver** can perform maximum ratio combining of "resolvable" multipath components
- less vulnerable to **jamming**
- signals can be **concealed**
- **many users can share bandwidth** without interfering much (CDMA)

Spread Spectrum (3)

Generating broadband pulses (direct sequence spread spectrum – DSSS):

- **divide symbol interval** in to N chip intervals $T_c = T/N$;
- form broadband pulse $h(t)$ by PAM modulating a
- **spreading sequence** $\{x_0, x_1, ..., x_{N-1}\}$
- using a **chip waveform** $h_c(t)$ at Nyquist rate $1/T_c$

$$h(t) = \sum_{m=0}^{N-1} x_m h_c(t - mT_c)$$

- the resulting pulse $h(t)$ has bandwidth of $h_c(t)$.

Orthogonal spreading sequences yield orthogonal pulses **(for CDMA)**

Spread Spectrum (4)

Inter-symbol-interference (ISI) and spread spectrum

- **Bandlimited signals** at Nyquist bandwidth have **large time-extent**!
  - Rectangular spectrum $\stackrel{\mathcal{F}}{\longleftrightarrow}$ **sinc**-pulse waveform
  - thus ISI can only be avoided for **flat channel**
- Spread pulses with large $N = 2WT$ are better localized in time:
  - zeros in sinc-function at $1/W = T/N \ll T$
  - thus *transmit pulse* comes **closer to being time-limited** to T
  - however *receive pulse* is affected by channel $\rightarrow$ matched filter output to be studied

Spread Spectrum (5)

Inter-symbol-interference (ISI) and spread spectrum (2)

- matched filter ($h^*(-t)$) output:

$$h(t) * c(t) * h^*(-t) = \rho_h(t) * c(t)$$

$c(t)$ ... channel impulse response
$\rho_h(t)$ ... pulse ACF

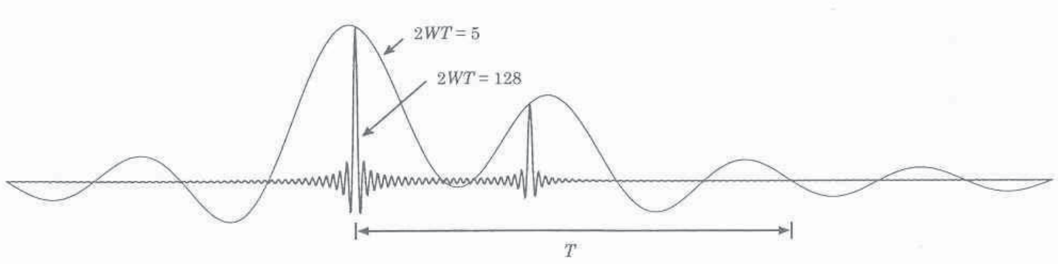


Fig. 6-20. The isolated pulses at the output of a matched filter for a two-path multipath channel with $\tau = 0.4T$ and $\alpha = 0.5$. Two bandwidth expansions are shown: $2WT = 5$ and $2WT = 128$. Note that the ISI will be small for $2WT = 128$ as long as delay spread τ is a little less than the symbol interval T, but for $2WT = 5$ ISI will be significant even for very small τ.

Spread Spectrum (6)

Spread Spectrum and Jamming or Interference

- Assume, jammer produces white signal of power P_J over bandwidth $W \rightarrow$ PSD $N_0/2 = P_J/(2W)$
- SNR at receiver input

$$\mathrm{SNR} = \frac{P}{N_0 W} = \frac{P}{P_J}$$

- BER P_b depends on SNR at matched filter output
  - this depends on E_b/N_0 only! independent of W!
  - noise power decreases with $2WT$ (nb. of signal dimensions) $\rightarrow P_b$ decreases
- $2WT$ is called **processing gain**

Spread Spectrum (7)

Spread Spectrum and Jamming or Interference

- One-dimensional jamming signal could be
  - **in direction of** $h(t) \rightarrow$ 100 % interference power; **no** processing gain
  - **orthogonal to** $h(t) \rightarrow$ 0 % interference power; **infinite** processing gain
- examples for one-dimensional jamming signals?
- jammer could be another user's signal $\rightarrow$ **code-division multiple access (CDMA)**

CDMA

- Each user transmits, using its own pulse shape $g_n(t)$